Intervention Protocol Establish Divide Fractions
Student:
Grade:

Teacher:

Class name: Date: 12/12/2019

This intervention is designed to build accuracy in dividing fractions. Requires 15 to 20 minutes each day.

Materials Needed: digital count-down timer, this intervention packet (protocol, progress monitoring chart, and weekly follow-up assessment probes), and a strip of cardstock or paper to cover the "answer column."

### Intervene: (complete these steps every day)

Sit with child in a quiet corner of the classroom. Use a strip of cardstock or paper to cover the column labeled "Answer."

Say, We are going to practice dividing fractions. When we divide, it is the same as solving for a missing factor. So 15 divided by 3 is the same as asking, 'What number times 3 equals 15?'

With fractions we are solving the same way: 1/3 divided by 2/3 is the same as asking what number times 2/3 equals 1/3. To solve this problem,  $2/3 \times 1/3$  we have to "undo" the 2/3 by multiplying both sides by 3/2 which give us unknown =  $3/2 \times 1/3$ .

Let's work the first one together. The top number of the fraction is called the numerator (point) and the bottom number in the fraction is called the denominator (point). To divide fractions we multiply the first numerator (point) by the second denominator (point) and write the answer in the numerator position.

Then we multiply the first denominator (point) by the second numerator (point) and write the answer in the denominator position. When we work with fractions, we always simplify the final fraction to its simplest form. Now we can compare our answer to see if we got it right.

Slide down the card covering the answer column to show the correct answer and ask the child, Is the answer the same? Yes, you got it right! Show the child how to make a check mark in the box for a correct match

- Set the timer for 10 minutes. Tell the child to begin working problems.
- Ensure that the child answers each problem correctly. Ensure that the child writes the number in the box without peeking at the answer box. Ensure that the child lifts the cover to check his or her answer.
- Ensure that the child makes a check mark in the "Match" box for correct answers.
- □ If there is not a match (the child's response was incorrect), guide the student to work the problem again and assist as needed to ensure correct problem solving.
- When the timer rings, tell the student to stop working.
- Count the number of correctly completed problems. Write this number on the Progress Monitoring Chart.
- Allow the child to select a small reward from the treasure chest for beating his or her last best score.

Teacher: \_

Class name: Date: 12/12/2019

# Troubleshoot

Students will make more rapid progress in learning this skill if they have mastered multiplication and division of whole numbers and especially fact families with multiplication and division. Because this intervention is selected based on student assessment, a student matched with this intervention should have demonstrated mastery of multiplication and division of whole numbers and fact families for multiplication and division.

The purpose of timing the intervention period is to contain the intervention to a focused and productive 10-minute period. This intervention requires direct assistance from the teacher. The teacher should sit beside the child and actively monitor each response to ensure the student is completing each problem accurately.

If a mistake is made, the teacher should guide the student to "try again" and provide prompts as needed to ensure correct responding. For example, the teacher might say, **Stop. What is the product of the first numerator and second denominator?** 

If the child cannot immediately correct the error, say, **Remember, we multiply the first numerator by the second denominator and write the answer here.** (point)

Guide the child to correctly respond following the sample script each time an error is made. Errors should rapidly decrease across sessions.

If one of the numbers is a whole number, remind the child how to represent a whole number as a fraction (with a I as the denominator).

If the student has trouble remembering the steps, have the student write each step of the problem so 3/5 divided by  $10 = (3 \times 1)/(5 \times 10) = 3/50$ .

## **Build Conceptual Understanding**

Complete several of these each day with the child, encouraging the child to solve each problem aloud:

When we divide a whole number by a fraction, the quotient is almost always greater than the dividend, which is different from what we are used to in working with whole numbers. Let's see why this is so. Let's divide 6 by 1/2.

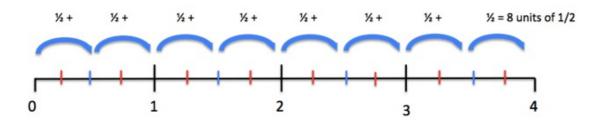
You start with 6	1	1	2	2	:	3	4	4	5	5	6	5
You divide each of 6 into two ½ parts	1/2	1/2	1/2	1/2	1/2	1/2	1/2	1/2	1/2	1/2	1/2	1/2
Count ½ parts in 6	1	2	3	4	5	6	7	8	9	10	11	12

## There are 12 1/2-parts in 6, so 6 divided by 1/2 is 12. Now, let's try dividing 6 by 1/3.

You start with 6		1			2			3			4			5			6	
You divide each of 6 into three 1/3 parts	1/3	1/3	1/3	1/3	1/3	1/3	1/3	1/3	1/3	1/3	1/3	1/3	1/3	1/3	1/3	1/3	1/3	1/3
Count 1/3 parts in 6	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

## There are 18 1/3-parts in 6, so 6 divided by 1/3 is 18.

Let's try it on a number line. Let's try  $4 \div 1/2$ .

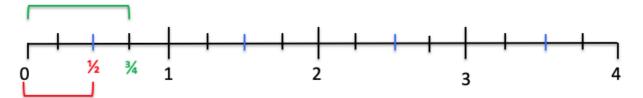


There are eight 1/2-parts in 4, so 4 divided by 1/2 is 8.

Now, let's try dividing a fraction by a fraction.

Let's think about  $1/2 \div 3/4$ .

½÷¾

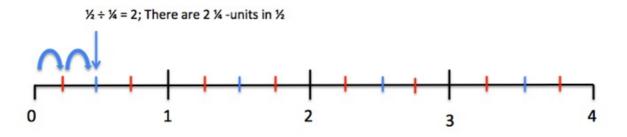


How many  $\frac{3}{4}$  parts are in  $\frac{1}{2}$ ? There is less than one  $\frac{3}{4}$  part in  $\frac{1}{2}$ . How do we know?

 $\frac{1}{2} = \frac{1}{4} + \frac{1}{4}$ 

 $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ . There are less than three  $\frac{1}{4}$ -parts in two- $\frac{1}{4}$  parts.

Let's try 1/2 divided by 1/4.



We have learned that when we divide with whole numbers, it is the same as solving for an unknown factor. In other words, 8 divided by 4 is the same as asking 4 times what number will equal 8? We could write this as '4 x ? = 8'. We can solve this in our heads because we know our multiplication facts. But we can also solve it procedurally.

You already know that any number can be multiplied by I without changing the quantity. Let's check and see.

8 x 4 = 32

8 (I) x 4 (I) = 32

This is useful to you because when you need to create an equivalent fraction with a different denominator (for example, when you want to compare close fraction quantities or when you want to add or subtract with fractions), you can multiply the fraction by a value of I to change the denominator.

Here is the problem: 3/4 + 5/6 =

Multiply each fraction by I: 3/4 (6/6) + 5/6 (4/4) =

To get equivalent fractions with same denominator: 18/24 + 20/24 =

Add to solve: 38/24 =

Then simplify fraction. I 14/24 = I 7/12

If you are really thinking hard, you can find a better common denominator.

Here is the problem again: 3/4 + 5/6 =

Multiply each fraction by I: 3/4 (3/3) + 5/6 (2/2) =

To get equivalent fractions with same denominator: 9/12 + 10/12 =

Add to solve: 19/12 =

Then simplify fraction: 17/12.

Is the answer the same? Yes, because we can multiply any value by I and it won't change the quantity.

Now, let's use a division problem. Remember, when we divide we are finding an unknown factor. In other words, when we have  $8 \div 4 = ?$ , we are asking '4 times what number will equal 8?'

So 1/4 ÷ 2/3 is the same as asking 2/3 times what number will equal 1/4 . We could write this like:

2/3y = 1/4 (If the 'y' bothers you, just use a question mark, for example, 2/3? = 1/4).

To solve, we will multiply 2/3 by a fraction that will cause the 2/3 to be the same as '1.' This step removes the 2/3 from one side of the equal sign, leaving the unknown value (y) by itself. If we multiply by 3/2 we will remove the 2/3 from the left side of the equal sign because 2/3 times Copyright 2019 Dr. Amanda M. VanDerHeyden. All rights reserved.

3/2 is 6/6 or '1.' Now, we have to do the same thing to the quantity on the other side of the equal sign.

2/3 (3/2) y = 1/4 (3/2) which becomes

 $y = 1/4 \times 3/2.$ 

Multiply to solve for y = 3/8.

We can check this as  $2/3 \times 3/8 = 6/24$  which simplifies to 1/4. So, yes,  $1/4 \div 2/3 = 3/8$ .

Now you know WHY 'invert and multiply' works when you divide fractions.

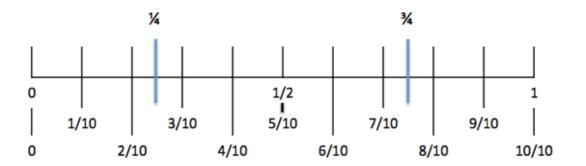
Let's try  $1/4 \div 1/10$  (How many 1/10 units are in 1/4)? We can count 1/10 units up to the 1/4 mark on the number line. It looks like about 2-1/2 1/10th units, but we can check and see.

We can write this problem as 1/4 ÷ 1/10 which is asking us 1/10 times what quantity will equal 1/4.

1/10y = 1/4

1/10(10/1)y = 1/4(10/1)

y = 10/4, which equals 2-1/2, so we were right. There are 2-1/2 1/10th units in 1/4.



Ask the student, With fractions, if the numerator stays the same and the denominator is made smaller, is the fraction quantity greater or lesser? In other words is 1/7 less than or greater than 1/6? Is 1/10 less than or greater than 1/2? Let's locate each fraction on a number line to check.

First, say aloud in your own words what each expression means. Then, fill in the blanks with < (lesser than) or > (greater than) to complete the statement. I'll show you how with the first one.

 $10 \div 10$  \_\_\_\_\_  $1/2 \div 4$ . The first quantity is 10 divided by 10 which is 1. That's easy. The second quantity is 1/2 divided by 4, so we are dividing the quantity of 1/2 into 4 equal parts. The answer will be 1/8. We can express this solution as  $1/8 \times 4 = 1/2$ . Or we can express this quantity as 1/8 + 1/8 + 1/8 = 1/2. We know that 1 is greater than 1/8, so we will insert the > in the blank to make the statement true.

Now it's your turn. Be sure to think aloud while you work and explain each step of your thinking to your teacher. What is each expression asking and how can you solve it?

I/8 x 3/8 is asking us to solve \_\_\_\_\_\_ sets of \_\_\_\_\_\_. The answer will be (greater than or less than) 3/8.

 $2/3 \div 1/4$  is asking us how many sets of \_\_\_\_\_\_ are in \_\_\_\_\_. We can write this as \_\_\_\_\_\_ • ? = 2/3. The answer will be (greater than or less than) 1.

3/4 x 1/2 is asking us to calculate \_\_\_\_\_\_ sets of \_\_\_\_\_. The answer will be (greater than or less than) 1.

5/6 x 2/10 is asking us to calculate \_\_\_\_\_\_ sets of \_\_\_\_\_\_. The answer will be (greater than or less than) 1

 $4/5 \div 1/5$  is asking us how many sets of \_\_\_\_\_ are in \_\_\_\_\_. We can write this as \_\_\_\_\_•? = \_\_\_\_. The answer will be (greater than or less than) 1.

 $1/10 \div 1/5$  is asking us how many sets of \_\_\_\_\_\_ are in \_\_\_\_\_. We can write this as \_\_\_\_\_ •? = \_\_\_\_. The answer will be (greater than or less than) 1.

 $7/8 \div 3/8$  is asking us how many sets of \_\_\_\_\_\_ are in \_\_\_\_\_. We can write this as \_\_\_\_\_\_  $\bullet$ ? = \_\_\_\_\_. The answer will be (greater than or less than) 1.

 $1/8 \div 3/8$  is asking us how many sets of \_\_\_\_\_\_ are in \_\_\_\_\_. We can write this as \_\_\_\_\_\_  $\bullet$ ? = \_\_\_\_\_. The answer will be (greater than or less than) 1.

Let's solve each division problem below.

1/12 ÷ 1/8

Rewrite as a multiplication problem with an unknown factor:  $1/8 \cdot y = 1/12$ 

Multiply the known factor quantity by a fraction that will cause the fraction to equal  $k 8/1 \cdot 1/8 = 8/8$  or 1.

Multiply the other side of the equation by the same quantity: $y = 1/12 \cdot 8/1$ .	
<b>Solve:</b> y = 8/12.	
Simplify: 2/3	
Now it's your turn.	
2/11 ÷ 9/3	
Rewrite as a multiplication problem with an unknown factor:	
Multiply the known factor quantity by a fraction that will cause the fraction to equal 1:	
Multiply the other side of the equation by the same quantity:	
Solve:	
Simplify:	
2/8 ÷ 7/6	
Rewrite as a multiplication problem with an unknown factor:	
Multiply the known factor quantity by a fraction that will cause the fraction to equal 1:	
Multiply the other side of the equation by the same quantity:	
Solve:	
Simplify:	
2/6 ÷ 8/6	
Rewrite as a multiplication problem with an unknown factor:	
Multiply the known factor quantity by a fraction that will cause the fraction to equal 1:	
Multiply the other side of the equation by the same quantity:	
Solve:	
Simplify:	
9/4 ÷ 9/12	
Rewrite as a multiplication problem with an unknown factor:	
Multiply the known factor quantity by a fraction that will cause the fraction to equal 1:	
Multiply the other side of the equation by the same quantity:	
Solve:	

Simplify:

6/10 ÷ 9/8

Rewrite as a multiplication problem with an unknown factor:

Multiply the known factor quantity by a fraction that will cause the fraction to equal 1: \_\_\_\_\_

Multiply the other side of the equation by the same quantity:

Solve:

Simplify: \_\_\_\_

Can 4/10 be written as a decimal?

Can 45/100 be written as a decimal?

Can you think of an easy way to convert 2/5 to a decimal? (Answer: change the denominator to 10ths)

- .45 is (less than or greater than) 1/2.
- .99 is (less than or greater than) 1.

.35 is (less than or greater than) 1/2.

.75 is (less than or greater than) 1/2.

How can you tell that a fraction is greater than 1? (i.e., the numerator is greater than the denominator) Ask the student to draw an example for you, using one of the number lines above.

When we multiply fractions does it matter which number comes first in the problem? Let's check and see. Do we get the same answer? Yes, so the product is the same regardless of which fraction appears first. The same is true when multiplying whole numbers.  $3 \times 5 = 5 \times 3$ . What about division?

Does it matter which number appears first in the problem? Is 2/5 divided by 3 equivalent to 3 divided by 2/5? Let's check and see. In division, the order of the numbers matters.

Monitoring	Student	Progress
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	CHART FOR
	Weekly Goal:
DAY I	My best score is:
	My score on the timed test is:
	Did I beat my score?
DAY 2	My best score is:
	My score on the timed test is:
	Did I beat my score?
DAY 3	My best score is:
	My score on the timed test is:
	Did I beat my score?
	Markensterne in
DAY 4	My best score is:
	My score on the timed test is:
	Did I beat my score?
DAY 5	My best score is:
DATS	
	My score on the timed test is:
	Did I beat my score?

Work Problem	Check Answer	Match?
$\frac{5}{11} \div \frac{10}{12} =$	$\frac{5}{11} \div \frac{10}{12} = \frac{6}{11}$	
$\frac{2}{10} \div \frac{10}{11} =$	$\frac{2}{10} \div \frac{10}{11} = \frac{11}{50}$	
$\frac{7}{11} \div \frac{7}{11} =$	$\frac{7}{11} \div \frac{7}{11} = 1$	
$\frac{2}{4} \div \frac{2}{10} =$	$\frac{2}{4} \div \frac{2}{10} = 2\frac{1}{2}$	
$\frac{7}{9} \div \frac{1}{8} =$	$\frac{7}{9} \div \frac{1}{8} = 6\frac{2}{9}$	
$\frac{3}{5} \div \frac{3}{10} =$	$\frac{3}{5} \div \frac{3}{10} = 2$	

Work Problem	Check Answer	Match?
$\frac{9}{10} \div \frac{6}{12} =$	$\frac{9}{10} \div \frac{6}{12} = 1\frac{4}{5}$	
$\frac{4}{12} \div \frac{7}{10} =$	$\frac{4}{12} \div \frac{7}{10} = \frac{10}{21}$	
$\frac{3}{12} \div \frac{10}{11} =$	$\frac{3}{12} \div \frac{10}{11} = \frac{11}{40}$	
$\frac{5}{9} \div \frac{4}{11} =$	$\frac{5}{9} \div \frac{4}{11} = 1\frac{19}{36}$	
$\frac{1}{2} \div \frac{5}{11} =$	$\frac{1}{2} \div \frac{5}{11} = 1\frac{1}{10}$	
$\frac{1}{6} \div \frac{1}{7} =$	$\frac{1}{6} \div \frac{1}{7} = 1\frac{1}{6}$	

Work Problem	Check Answer	Match?
$\frac{4}{7} \div \frac{5}{11} =$	$\frac{4}{7} \div \frac{5}{11} = 1\frac{9}{35}$	
$\frac{9}{11} \div \frac{8}{11} =$	$\frac{9}{11} \div \frac{8}{11} = 1\frac{1}{8}$	
$\frac{3}{11} \div \frac{11}{12} =$	$\frac{3}{11} \div \frac{11}{12} = \frac{36}{121}$	
$\frac{6}{7} \div \frac{7}{11} =$	$\frac{6}{7} \div \frac{7}{11} = 1\frac{17}{49}$	
$\frac{7}{10} \div \frac{7}{11} =$	$\frac{7}{10} \div \frac{7}{11} = 1\frac{1}{10}$	
$\frac{9}{12} \div \frac{10}{12} =$	$\frac{9}{12} \div \frac{10}{12} = \frac{9}{10}$	

Work Problem	Check Answer	Match?
$\frac{5}{6} \div \frac{1}{2} =$	$\frac{5}{6} \div \frac{1}{2} = 1\frac{2}{3}$	
$\frac{3}{6} \div \frac{9}{12} =$	$\frac{3}{6} \div \frac{9}{12} = \frac{2}{3}$	
$\frac{1}{2} \div \frac{9}{12} =$	$\frac{1}{2} \div \frac{9}{12} = \frac{2}{3}$	
$\frac{1}{11} \div \frac{10}{11} =$	$\frac{1}{11} \div \frac{10}{11} = \frac{1}{10}$	
$\frac{2}{6} \div \frac{9}{11} =$	$\frac{2}{6} \div \frac{9}{11} = \frac{11}{27}$	
$\frac{1}{2} \div \frac{2}{3} =$	$\frac{1}{2} \div \frac{2}{3} = \frac{3}{4}$	

Work Problem	Check Answer	Match?
$\frac{5}{12} \div \frac{6}{10} =$	$\frac{5}{12} \div \frac{6}{10} = \frac{25}{36}$	
$\frac{3}{5} \div \frac{2}{7} =$	$\frac{3}{5} \div \frac{2}{7} = 2\frac{1}{10}$	
$\frac{6}{8} \div \frac{6}{10} =$	$\frac{6}{8} \div \frac{6}{10} = 1\frac{1}{4}$	
$\frac{1}{3} \div \frac{1}{2} =$	$\frac{1}{3} \div \frac{1}{2} = \frac{2}{3}$	
$\frac{1}{2} \div \frac{8}{11} =$	$\frac{1}{2} \div \frac{8}{11} = \frac{11}{16}$	
$\frac{3}{5} \div \frac{9}{11} =$	$\frac{3}{5} \div \frac{9}{11} = \frac{11}{15}$	

Teacher	
Grade: _	

Student:

<b>Divide Fraction</b>
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Date:	12/12/2019	(version:	50f3)
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"These problems are division with fractions. Let's do the first one together." Work the first problem with the student(s) to make sure student(s) understand the task. "When I say begin, start with the second problem on the first row and work across. Don't skip any problems and be sure to simplify your answer. Work as many problems as you can before the time is up. Do you have any questions?" Set timer for 2 minutes, start timer, and say, "Begin." Stop student after 2 minutes and count up problems correct.

$\begin{array}{c} sample $	$\frac{3}{4} \div \frac{2}{3} =$	$\frac{6}{10} \div \frac{2}{12} =$	$\frac{2}{6} \div \frac{3}{12} =$	$\frac{1}{11} \div \frac{11}{12} =$	( )/4
$\frac{4}{7} \div \frac{8}{12} =$	$\frac{1}{10} \div \frac{2}{12} =$	$\frac{2}{3} \div \frac{2}{3} =$	$\frac{1}{3} \div \frac{3}{10} =$	$\frac{6}{11} \div \frac{7}{11} =$	( )/9
$\frac{3}{11} \div \frac{10}{11} =$	$\frac{1}{2} \div \frac{3}{11} =$	$\frac{4}{11} \div \frac{10}{11} =$	$\frac{2}{9} \div \frac{1}{6} =$	$\frac{2}{3} \div \frac{7}{11} =$	( )/14
$\frac{1}{2} \div \frac{4}{9} =$	$\frac{4}{12} \div \frac{3}{12} =$	$\frac{1}{3} \div \frac{1}{3} =$	$\frac{1}{8} \div \frac{5}{8} =$	$\frac{9}{12} \div \frac{11}{12} =$	( )/19
$\frac{2}{10} \div \frac{10}{11} =$	$\frac{2}{8} \div \frac{2}{11} =$	$\frac{4}{11} \div \frac{9}{10} =$	$\frac{6}{9} \div \frac{7}{12} =$	$\frac{1}{3} \div \frac{3}{4} =$	( )/24
$\frac{4}{11} \div \frac{10}{12} =$	$\frac{3}{7} \div \frac{1}{2} =$	$\frac{1}{10} \div \frac{4}{11} =$	$\frac{7}{12} \div \frac{5}{10} =$	$\frac{1}{6} \div \frac{7}{12} =$	( )/29

Total problems correct:\_\_

**Divide Fractions Answer Key** 

Version: 50f3

$\frac{7}{10} \div \frac{10}{12} = \frac{21}{25}$	$\frac{3}{4} \div \frac{2}{3} = 1\frac{1}{8}$	$\frac{6}{10} \div \frac{2}{12} = 3\frac{3}{5}$	$\frac{2}{6} \div \frac{3}{12} = 1\frac{1}{3}$	$\frac{1}{11} \div \frac{11}{12} = \frac{12}{121}$
$\frac{4}{7} \div \frac{8}{12} = \frac{6}{7}$	$\frac{1}{10} \div \frac{2}{12} = \frac{3}{5}$	$\frac{2}{3} \div \frac{2}{3} = 1$	$\frac{1}{3} \div \frac{3}{10} = 1\frac{1}{9}$	$\frac{6}{11} \div \frac{7}{11} = \frac{6}{7}$
$\frac{3}{11} \div \frac{10}{11} = \frac{3}{10}$	$\frac{1}{2} \div \frac{3}{11} = 1\frac{5}{6}$	$\frac{4}{11} \div \frac{10}{11} = \frac{2}{5}$	$\frac{2}{9} \div \frac{1}{6} = 1\frac{1}{3}$	$\frac{2}{3} \div \frac{7}{11} = 1\frac{1}{21}$
$\frac{1}{2} \div \frac{4}{9} = 1\frac{1}{8}$	$\frac{4}{12} \div \frac{3}{12} = 1\frac{1}{3}$	$\frac{1}{3} \div \frac{1}{3} = 1$	$\frac{1}{8} \div \frac{5}{8} = \frac{1}{5}$	$\frac{9}{12} \div \frac{11}{12} = \frac{9}{11}$
$\frac{2}{10} \div \frac{10}{11} = \frac{11}{50}$	$\frac{2}{8} \div \frac{2}{11} = 1\frac{3}{8}$	$\frac{4}{11} \div \frac{9}{10} = \frac{40}{99}$	$\frac{6}{9} \div \frac{7}{12} = 1\frac{1}{7}$	$\frac{1}{3} \div \frac{3}{4} = \frac{4}{9}$
$\frac{4}{11} \div \frac{10}{12} = \frac{24}{55}$	$\frac{3}{7} \div \frac{1}{2} = \frac{6}{7}$	$\frac{1}{10} \div \frac{4}{11} = \frac{11}{40}$	$\frac{7}{12} \div \frac{5}{10} = 1\frac{1}{6}$	$\frac{1}{6} \div \frac{7}{12} = \frac{2}{7}$